DOCUMENT RESUME

ED 261 858 SE 045 982

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TITLE Exploring High School Mathematics with Logo.
PUB DATE 85
NOTE 13p.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Computer Oriented Programs; *Computer Software; *Geometric Concepts; High Schools; *Learning Activities; *Mathematics Instruction; Microcomputers; *Programing; *Secondary School Mathematics; Teaching Methods
IDENTIFIERS *Logo Programing Language

ABSTRACT

Having students draw and investigate geometric shapes in a Logo environment is advocated as an excellent way to have them constructively learn geometric concepts. This document illustrates the potential of Logo for getting high school students involved in mathematical explorations by presenting a sequence of seven activities. The activities concern: drawing geometric figures with the Logo turtle, drawing more complicted figures, exploring regular polygons, investigating rotational symmetry, investigating regular star polygons, discovering methods for drawing circles with the turtle, and using Logo to explore coordinate geometry and the Pythagorean Theorem. (MNS)

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EXPLORING HIGH SCHOOL MATHEMATICS WITH LOGO

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Imagine that you have control of a small triangular creature called a turtle that lives on a computer display screen. The turtle responds to a several movement commands: FORWARD moves the turtle a given number of units in the direction it is facing. RIGHT rotates the turtle in a clockwise direction a given number of degrees. BACK moves the turtle backward. LEFT turns the turtle counterclockwise. When the turtle has been given the PENDOWN command, it leaves a trace of the path it follows. The PENUP command allows the turtle to move without leaving a trace. Given that the turtle understands these and other commands, it is easy to see how the turtle can be made to draw a great variety of pictures and geometric shapes. Such "turtle graphics" are an essential part the Logo programming language and will be the major topic of discussion in this article.

Plane geometry, as it is taught in school mathematics, is essentially the study of figures that exist within a mathematical plane. The computer screen on which the Logo turtle leaves its trace is an excellent representation of a small portion of such a plane. Thus, it has been suggested in the literature that having students investigate and explore the world of turtle graphics is an excellent way to have them learn about school geometry (Billings, 1983; Billstein, 1982). As Abelson and diSessa have noted, with Logo, students can "regard plane-geometric figures not as static entities but as tracings made on a display screen by a

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computer-controlled 'turtle' " (1980, p. xiv). "We can think of turtle commands as a way to draw geometric figures" (Abelson and diSessa, 1980, p. 11).

Thus, it seems that having students draw and investigate geometric shapes in a Logo environment is an excellent way to have them constructively learn geometric concepts. That is, if students attempt to direct the turtle to draw a rectangle, for example, they must do far more than conjure up a mental image of a rectangle, as they would if they drew it freehand. They must make their conceptualization of rectangle explicit enough to develop a set of move commands that can be entered into the computer. By entering these commands into the computer, they test the validity of their conceptualization. In this way, the students' conceptualization is expressed in a way that allows it to be actively manipulated, modified, and studied. Alternately, if students are given a Logo procedure that will quickly draw regular star polygons when the proper number of sides and angle of turn are input, the students will be able to investigate the mathematical relationship that exists between these inputs more easily than would be possible if they had to draw the figures themselves. Furthermore, the computer seems to have a motivational effect on students. Many times students can develop geometric concepts by investigating inherently "neat" phenomena such as a Logo procedures that draw spirals or many-pointed stars (Papert, 1980).

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The purpose of this article is to illustrate the potential of Logo for getting high school students involved in mathematical explorations. A sequence of activities will be given to illustrate this use.

ACTIVITY 1: Drawing Geometric Figures with the Turtle

After you have familiarized your students with the basic movement commands in Logo, ask them to direct the turtle to draw some common geometric shapes such as an equilateral triangle, two parallel line segments cut by a transversal, a parallelogram that is neither a rectangle nor a rhombus, a right triangle, or two congruent triangles. An equilateral triangle, for example, can be drawn with the following sequence of commands:

```
FORWARD 50
RIGHT 120
FORWARD 50
RIGHT 120
FORWARD 50
```

However, even the simplest geometric figure can be challenging to draw when one is restricted to the FORWARD and RIGHT commands. For instance, you might want to challenge your trigonometry students to direct the turtle to draw a triangle with sides of length 50 and 70 and included angle of 40 degrees.

Teaching the Turtle New Commands

Although it is great fun to make drawings with the turtle, it can become somewhat tedious and repetitive to give the turtle a long sequence of commands. Fortunately, there is a way to teach the turtle new

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commands. These new commands can then be used to have the turtle draw more complicated and exciting figures. For example, in order to teach the turtle how to draw a square, we give the following sequence of commands:

```
TO SQUARE
   FORWARD 100
   RIGHT 90
   FORWARD 100
   RIGHT 90
   FORWARD 100
   RIGHT 90
   FORWARD 100
END
```

or, more briefly,

```
TO SQUARE
   REPEAT 4 [FD 100 RT 90]
END
```

The first line in this procedure tells the turtle the name of the new command, SQUARE. The rest of the procedure tells the turtle the sequence of instructions to follow whenever the command SQUARE is given. After teaching the turtle this new command, whenever we want the turtle to draw a square with sides of length 100, we simply give the turtle the SQUARE command. We can even introduce variables or inputs into our procedures. For example, the procedure below will draw a square with sides of length :S. (After the procedure has been defined, one has to type SQUARE 75, for instance, to get a square with sides of length 75.)

```
TO SQUARE :S
   REPEAT 4 [FD :S RT 90]
END
```

ACTIVITY 2: Similar Figures

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We can easily explore the concept of similarity using procedures with inputs. By inputting different values for :S in the procedure below, we will get a set of similar rectangles. Once students have seen the figures drawn by this procedure, they can be asked to prove that the rectangles drawn are similar.

```
TO RECTANGLE :S
   FD :S*60 RT 90
   FD :S*40 RT 90
   FD :S*60 RT 90
   FD :S*40 RT 90
END
```

ACTIVITY 3: Regular Polygons

Another interesting activity involves students in exploring regular polygons. Have them write procedures that teach the turtle to draw a regular pentagon, a regular hexagon, and a regular octagon. Challenge students to write a procedure that takes a number as "input" then draws a regular polygon with the given number of sides. (This activity is an excellent chance to try your hand at involving students in "discovery" learning with Logo.)

ACTIVITY 4: Symmetry

Logo can be used to have students investigate the idea of rotational symmetry of figures. Example: Have students type in either one of the procedures below --one procedure teaches the turtle how to draw FIG1, the other teaches the turtle how to draw FIG3. Now, with the turtle facing straight up, have it draw one of the

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figures. Then turn the turtle to the right the number of degrees that you think it takes to get a rotational symmetry of the figure and have the turtle draw the figure again. If the turned figure coincides with the original, the angle of rotation you specified is a rotational symmetry for the figure.

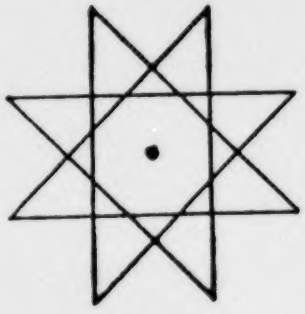
```
TO FIG1
  PU FD 43.5 RT 126 PD
  REPEAT 5 [FD 50 RT 72]
  PU HOME PD
END
```

```
TO FIG3
  PU FD 64 RT 157.5 PD
  REPEAT 8 [FD 118 RT 135]
  PU HOME PD
END
```

ACTIVITY 5: Regular Star Polygons

A regular star polygon is a nonsimple closed curve consisting only of line segments and having all of its sides the same length and all of its angles the same measure. (See Figure 1.) Having students investigate regular star polygons with the student activity below is an excellent way to involve them in mathematical problem solving. Be sure to have students organize and analyze all the data they collect.

Fig. 1



Before you start students on the activity, demonstrate the method used to draw a regular star polygon with 9 sides. First, we place 9 points equally

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spaced around a circle. Then we pick another number, in this case 2. We then draw line segments to every second point. We stop when we return to the starting point. Since all 9 points on the circle were included in the figure drawn, we call the figure the regular star polygon (9,2). Demonstrate that if we attempt to draw the regular star polygon (9,3), we get an equilateral triangle, not a star polygon.

The objective of the activity is to have students discover two things about regular star polygons. First, the regular star polygons (N,D) and (N,K) are congruent if and only if D+K=N. Second, the regular star polygon (N,K) exists if and only if N and K are relatively prime, that is, the fraction N/K is irreducible. Another discovery students should make about POLY is that if K=1 or K=N-1, then POLY N K draws a regular polygon of N sides.

STUDENT ACTIVITY

Type the procedure below into the computer. You can use this procedure to make the turtle attempt to draw regular star polygons. For example, you can have the turtle attempt to draw the regular star polygon (10,3) by typing POLY 10 3. Sometimes the figure drawn is a regular star polygon (see POLY 10 3); other times the figure drawn is not a regular star polygon (see POLY 10 2). We can use the POLY procedure to discover which regular star polygons exist.

```
TO POLY :N :K
 REPEAT :N [RT :K*360/:N FD 50]
END
```

Try to determine which regular 8-sided star polygons exist. ((8,3) and (8,5)) How many distinct (noncongruent) ones are there? Use the

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POLY procedure to determine how many 10- and 12-sided regular star polygons exist. Now try to find the numerical relationship that must exist between N and K in order for the regular star polygon (N,K) to exist. That is, try to find a way to predict whether the regular star polygons (50,9), (30,7), and (32,9) exist <u>without using the computer.</u> Use the POLY procedure to check you answers. Also, how must K and D be related in order for the regular star polygons (N,K) and (N,D) to be congruent? Use the relationship you discover to predict how many distinct 20-sided regular star polygons exist.

Some high school geometry texts contain proofs that every regular polygon has a circumscribed circle. An extension of this proof can be used to show that the Logo POLY procedure draws stars whose vertices lie on a circle. Challenge your students to devise such a proof.

Another version of POLY is found in most Logo books.

```
TO POLY :N :A
   REPEAT :N (RT :A FD 50)
END
```

In this version students should be asked to determine how the number of sides :N and the angle of turn :A must be related in order for POLY to draw a regular polygon. They should also be asked to discover how the number of sides and angle of turn must be related in order for POLY to draw a figure whose starting point and ending point coincide. This version of POLY is helpful in relating POLY explorations to theorems about the measures of the vertex and exterior angles of regular polygons. You can also ask students to determine if any of these theorems can be extended (or modified) to include regular star polygons.

<u>ACTIVITY 6: Circles</u>

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How can you get the turtle to draw a circle? Many students discover the following procedure.

```
TO CIRCLE1
  REPEAT 360 [FD 1 RT 1]
END
```

Once students have discovered this procedure, ask them to write a procedure to draw a smaller circle? a faster circle? Challenge them to write a procedure that will draw a circle with a given radius? Ask them if the figure drawn is really a circle?

After students have discovered that the above procedure does not draw a circle, but rather a regular polygon, challenge them to find another method for drawing a circle. Many will discover the following:

```
TO CIRCLE2
  REPEAT 360 [PU FD 30 PD FD 1 PU BK 31 RT 1]
END
```

Note that this method seems more consistent with the usual definition of circle as a locus of points.

ACTIVITY 7: Coordinate Geometry and the Pythagorean Theorem

The SETXY command positions the turtle at a specified point on the screen according to a Cartesian coordinate system. Thus, students can be asked to have the turtle draw various geometric figures using this command. Another good activity is to have the students write a procedure that will take as input the length and width of a rectangle, then will draw a rectangle that has

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the given length and width, the origin as its center, and its diagonals drawn in. Ask students to notice where the diagonals intersect.

CONCLUSION

Many other concepts can be explored with Logo. Slides, flips, and turns from transformational geometry, and the distance formula for points in the plane are two additional examples. However, the goal of this article was not to provide an exhaustive listing of such concepts, but rather to illustrate the use of this powerful programming language as a resource in mathematics instruction. Hopefully, the reader has been convinced of the great potential for using Logo to get students actively engaged in mathematical exploration and problem solving.

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